Fje više promjenljivih

Def (granične vr(j)) Fja f ima $\lim_{x \to x_0} f(x)$, ako $\exists A$ td $\forall \varepsilon > 0$ $\exists \delta > 0$ td $\forall x$: $0 < |x - x_0| < \delta \rightarrow |x - x_0| = \sqrt{(x_1 - x_1^0)^2 + (x_2 - x_2^0)^2 + \dots + (x_m - x_m^0)^2}$ bude $|f(x) - A| < \varepsilon$.

1) Naći $\lim_{x \to 0} (\lim_{y \to \infty} f(x,y))$ i $\lim_{y \to \infty} (\lim_{x \to 0} f(x,y))$, ako: $f(x,y) = \frac{1}{xy} tg \frac{xy}{1+xy}$

(uzastopne gr. vrijednosti)

Rj $x \neq 0$ fiksirano, $\lim_{y \to +\infty} \frac{xy}{1+xy} = \lim_{y \to +\infty} \frac{x}{\frac{1}{y} + x} = 1$, pa zbog neprekidnosti $tg x$, važi: $\lim_{y \to \infty} \frac{1}{xy} tg \frac{xy}{1+xy} = 0 \cdot tg 1 = 0$.

Fiksirajmo y: važi $\lim_{x \to 0} \frac{tg x}{x} = \lim_{x \to 0} \frac{\sin x}{\cos x \cdot x} = 1$

$$\lim_{x \to 0} f(x,y) = \lim_{x \to 0} \frac{tg \frac{xy}{1+xy}}{\frac{xy}{1+xy}} \cdot (xy+1)^{-1} = \lim_{x \to 0} \frac{1}{xy+1} = 1$$

$$\lim_{x \to 0} \left(\lim_{y \to \infty} \frac{1}{xy} tg \frac{xy}{1+xy}\right) = \lim_{x \to 0} 0 = 0.$$

$$\lim_{y \to 0} \left(\lim_{x \to 0} \frac{1}{xy} tg \frac{xy}{1+xy}\right) = \lim_{y \to 0} 1 = 1$$

Def. (granične vrijednosti) - Hajne: Fja f ima $\lim_{x \to x_0} f(x)$ ako $\exists A \in R$ td. za proizvoljno $\{x_n\}$, $x_n \in E \setminus \{x_0\}$, $E \subseteq R^m$, koji konvergira ka x_0 odgovarajući $\{f(x_n)\}$ konvergira ka A.

Pokazati da je za fju $f(x,y) = \frac{x-y}{x+y}$, $\lim_{x \to 0} (\lim_{y \to 0} f(x,y)) = 1$, $\lim_{y \to 0} (\lim_{x \to 0} f(x,y)) = -1$ a $\lim_{\substack{x \to 0 \\ y \to 0}} f(x,y)$ ne postoji.

$$\lim_{y\to 0}\left(\lim_{x\to 0}\frac{x-y}{x+y}\right)=\lim_{y\to 0}\frac{-y}{y}=-1,\quad \lim_{x\to 0}\left(\lim_{y\to 0}\frac{x-y}{x+y}\right)=\lim_{x\to 0}\frac{x}{x}=1$$

Da li postoji $\lim_{\substack{x\to 0\\ y\to 0}} f(x,y)$?

$\{(\frac{1}{n},\frac{1}{n})\}\xrightarrow[n\to\infty]{}(0,0)$, a $\{(\frac{2}{n},\frac{1}{n})\}\xrightarrow[n\to\infty]{}(0,0)$

$$\left.\begin{aligned} f\left(\tfrac{1}{n},\tfrac{1}{n}\right)&=\frac{0}{\frac{2}{n}}=0\to 0,\ n\to\infty\\ f\left(\tfrac{2}{n},\tfrac{1}{n}\right)&=\frac{\frac{1}{n}}{\frac{3}{n}}=\frac{1}{3}\to\frac{1}{3},\ n\to\infty\end{aligned}\right\}\Rightarrow \nexists \lim_{\substack{x\to 0\\ y\to 0}} f(x,y).$$

2) Ispitati neprekidnost fje: $f(x,y)=\begin{cases}\dfrac{e^{xy}-1}{2^{xy}-1}\cdot\dfrac{\sin\sqrt{x^2y^2}}{\sqrt{x^2y^2}}, & (x,y)\neq(0,0)\\ 1, & (x,y)=(0,0)\end{cases}$

u tački $(x,y)=(0,0)$

Rj.

Za $(x,y)\neq(0,0)$ fja neprekidna kao kompozicija neprekidnih fja

Za $(x,y)=(0,0)$.

$$\lim_{\substack{x\to 0\\ y\to 0}}\frac{e^{xy}-1}{2^{xy}-1}\cdot\frac{\sin\sqrt{x^2y^2}}{\sqrt{x^2y^2}}=\lim_{\substack{x\to 0\\ y\to 0}}\frac{\frac{e^{xy}-1}{xy}}{\frac{2^{xy}-1}{xy}}\cdot\frac{\sin\sqrt{x^2y^2}}{\sqrt{x^2y^2}},$$

$x\to 0$, $y\to 0$, $t\to 0$

$$\lim_{t\to 0}\frac{e^t-1}{t}=1,\quad \lim_{t\to 0}\frac{a^t-1}{t}=\ln a,\quad \lim_{t\to 0}\frac{\sin t}{t}=1$$

$$\lim_{\substack{x\to 0\\ y\to 0}}\frac{1}{\ln 2}\cdot 1=\frac{1}{\ln 2},\quad \lim_{\substack{x\to 0\\ y\to 0}} f(x,y)=f(0,0)\ \text{ a } \frac{1}{\ln 2}\neq 1\Rightarrow$$

F nije nep. u $(0,0)$

3) Naći graničnu vrijednost $\lim_{\substack{x\to+\infty \\ y\to+\infty}} \left(\frac{xy}{x^2+y^2}\right)^{x^2}$

Rj.

$x^2+y^2 \geq 2xy \Rightarrow \frac{xy}{x^2+y^2} \leq \frac{xy}{2xy} = \frac{1}{2}$

$(x-y)^2 \geq 0$

$x^2-2xy+y^2 \geq 0$

$x^2+y^2 \geq 2xy$

$0 < \left(\frac{xy}{x^2+y^2}\right)^{x^2} \leq \left(\frac{1}{2}\right)^{x^2} \to 0, \ x\to+\infty$

$\Rightarrow \lim_{\substack{x\to+\infty \\ y\to+\infty}} \left(\frac{xy}{x^2+y^2}\right)^{x^2} = 0$

1) Odrediti parcijalne izvode fja:

a) $u = \operatorname{arctg}\frac{x}{y}$

$\frac{\partial u}{\partial x} = \frac{1}{1+\left(\frac{x}{y}\right)^2} \cdot \left(\frac{x}{y}\right)'_x = \frac{1}{1+\left(\frac{x}{y}\right)^2} \cdot \frac{1}{y} = \frac{y^2}{y^2+x^2} \cdot \frac{1}{y}$

$\frac{\partial u}{\partial y} = \frac{1}{1+\left(\frac{x}{y}\right)^2} \cdot \left(\frac{x}{y}\right)'_y = \frac{y^2}{y^2+x^2} \cdot \frac{-x}{y^2} = -\frac{x}{y^2+x^2}$

b) $u = x^2\sqrt{y} + 7y$

$\frac{\partial u}{\partial x} = 2x\sqrt{y}, \quad \frac{\partial u}{\partial y} = x^2\frac{1}{2\sqrt{y}} + 7$

c) $u = \sin x^2 \cdot \cos\sqrt{y}$

$\frac{\partial u}{\partial x} = \cos x^2 \cdot 2x \cos\sqrt{y}$

$\frac{\partial u}{\partial y} = -\sin x^2 \cdot \sin\sqrt{y} \cdot \frac{1}{2\sqrt{y}}$

d) $u = x^x \cdot y^y$

$\frac{\partial u}{\partial x} = y^y \cdot (\ln x + 1)$

$x^x = m \ / \ln$

$x \ln x = \ln m$

$\ln x + x\frac{1}{x} = \frac{1}{m} \cdot m'$

$m' = x^x(1+\ln x)$

$\frac{\partial u}{\partial y} = x^x y^y(\ln y + 1)$

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e) $u = x^{yz} + y^{xz} + z^{xy}$

$$\frac{\partial u}{\partial x} = yz\,x^{yz-1} + y^{xz}\ln y \cdot z + z^{xy}\cdot \ln z \cdot y$$

$$\frac{\partial u}{\partial y} = x^{yz}\ln x \cdot z + xz\,y^{xz-1} + z^{xy}\cdot \ln z \cdot x$$

$$\frac{\partial u}{\partial z} = x^{yz}\cdot \ln x\, y + y^{xz}\cdot \ln y \cdot x + xy\, z^{xy-1}$$

2) Naći drugi parcijalne izvode $\frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial x \partial y}, \frac{\partial^2 f}{\partial y^2}$ za fje

$f(x,y) = \operatorname{arctg} \frac{y}{x}$

Rj:

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{1}{1+\frac{y^2}{x^2}}\cdot\left(\frac{-y}{x^2}\right)\right) = \frac{\partial}{\partial x}\left(\frac{x^2}{x^2+y^2}\left(\frac{-y}{x^2}\right)\right) =$$

$$= -y\,\frac{\partial}{\partial x}\left(\frac{1}{x^2+y^2}\right) = -y\cdot\frac{-2x}{(x^2+y^2)^2} = \frac{2xy}{(x^2+y^2)^2}$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial x}\left(\frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\frac{1}{x}\right) = \frac{\partial}{\partial x}\left(\frac{x^2}{x^2+y^2}\cdot\frac{1}{x}\right) =$$

$$= \frac{\partial}{\partial x}\left(\frac{x}{x^2+y^2}\right) = \frac{x^2+y^2-x(2x)}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{x}{x^2+y^2}\right) = \frac{-2xy}{(x^2+y^2)^2}$$

3) Naći totalni diferencijal fje $f(x,y) = \frac{xy}{x-y}$

Rj:

$$df = \frac{\partial f}{\partial x}\cdot dx + \frac{\partial f}{\partial y}\,dy$$

$$\frac{\partial f}{\partial x} = \frac{y(x-y)-xy}{(x-y)^2} = \frac{yx-y^2-xy}{(x-y)^2} = \frac{-y^2}{(x-y)^2}$$

$$\frac{\partial f}{\partial y} = \frac{x(x-y)+xy}{(x-y)^2} = \frac{x^2-xy+xy}{(x-y)^2} = \frac{x^2}{(x-y)^2}$$

$$\Rightarrow df = \frac{-y^2}{(x-y)^2}\,dx + \frac{x^2}{(x-y)^2}\,dy$$

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4) Približno izračunati $1{,}002 \cdot 2{,}003^2 \cdot 3{,}004^3$.

Rj. $f(x,y,z) = (1+x)(2+y)^2(3+z)^3$

$f(x_0+\Delta x, y_0+\Delta y, z_0+\Delta z) - f(x_0,y_0,z_0) \approx df(x_0,y_0,z_0)$

$\Delta x = 0{,}002,\ \Delta y = 0{,}003,\ \Delta z = 0{,}004$

$x_0 = y_0 = z_0 = 0$

$f(0{,}002;\ 0{,}003;\ 0{,}004) - f(0,0,0) \approx df(0,0,0)$

$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} \cdot dy + \frac{\partial f}{\partial z} \cdot dz$, $dx = \Delta x$, $dy = \Delta y$ | $f = xy^2z^3$

$\frac{\partial f}{\partial x} = (2+y)^2(3+z)^3$

$\frac{\partial f}{\partial y} = (1+x)\,2(2+y)(3+z)^3$

$\frac{\partial f}{\partial z} = (1+x)(2+y)^2\,3(3+z)^2$

$df(0,0,0) = ?$

$\frac{\partial f}{\partial x}(0,0,0) = 2^2 \cdot 3^3 = 4 \cdot 27 = 108$

$\frac{\partial f}{\partial y}(0,0,0) = 108;\quad \frac{\partial f}{\partial z}(0,0,0) = 108$

$f(1{,}002,\ 2{,}003,\ 3{,}004) =$

$= f(1,2,3) + df(1,2,3)$

$df = \frac{\partial f}{\partial x}(1,2,3)\cdot \Delta x + \frac{\partial f}{\partial y}(1,2,3)\Delta y +$

$+ \frac{\partial f}{\partial z}(1,2,3)\Delta z$.

$\Delta x = 0{,}002$

$\Delta y = 0{,}003$

$\Delta z = 0{,}004$

$df(0,0,0) = 108 \cdot 0{,}002 + 108 \cdot 0{,}003 + 108 \cdot 0{,}004 = 0{,}972$.

$f(0{,}002;\ 0{,}003;\ 0{,}004) = f(0,0,0) + df(0,0,0) = 108 + 0{,}972 = 108{,}972$

Diferencijal višeg reda

$u_{xx} = \frac{\partial^2 u}{\partial x^2},\quad u_{xy} = \frac{\partial^2 u}{\partial x \partial y},\quad u_{yy} = \frac{\partial^2 u}{\partial y^2}$

$d^2u = \frac{\partial^2 u}{\partial x^2} \cdot dx^2 + 2\frac{\partial^2 u}{\partial x \partial y} dx dy + \frac{\partial^2 u}{\partial y^2} \cdot dy^2$

$d^n u = \left(\frac{\partial u}{\partial x} \cdot dx + \frac{\partial u}{\partial y} \cdot dy\right)^n,\ n \in \mathbb{N}$

1) Za $u=\ln(x^2+y^2)$, dokazati $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$, odrediti du, d^2u)

Rj: $\frac{\partial u}{\partial x}=\frac{1}{x^2+y^2}\cdot 2x$, $\frac{\partial u}{\partial y}=\frac{1}{x^2+y^2}\cdot 2y$

$$\frac{\partial^2 u}{\partial x^2}=\frac{2(x^2+y^2)-2x\cdot 2x}{(x^2+y^2)^2}=\frac{2y^2-2x^2}{(x^2+y^2)^2}$$

$$\frac{\partial^2 u}{\partial x\partial y}=\frac{-2x2y}{(x^2+y^2)^2}$$

$$\frac{\partial^2 u}{\partial y^2}=\frac{2(x^2+y^2)-2y\cdot 2y}{(x^2+y^2)^2}=\frac{2x^2-2y^2}{(x^2+y^2)^2}$$

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{2y^2-2x^2+2x^2-2y^2}{(x^2+y^2)^2}=0$$

$$du=\frac{2x}{x^2+y^2}dx+\frac{2y}{x^2+y^2}dy$$

$$d^2u=\frac{2y^2-2x^2}{(x^2+y^2)^2}dx^2-2\frac{4xy}{(x^2+y^2)^2}dxdy+\frac{2x^2-2y^2}{(x^2+y^2)^2}dy^2$$

Izvod implicitno zadate fje

$F=F(x,y)$, $y=y(x)$

$$\frac{\partial F}{\partial x}+\frac{\partial F}{\partial y}\cdot\frac{\partial y}{\partial x}=0\Rightarrow\frac{\partial y}{\partial x}=-\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}}$$

$F=F(x,y,z)=0$, $z=z(x,y)$

$\frac{\partial z}{\partial x}=?$

$$\frac{\partial F}{\partial x}+\frac{\partial F}{\partial z}\cdot\frac{\partial z}{\partial x}=0\to\frac{\partial z}{\partial x}=-\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}}$$

$$\frac{\partial F}{\partial y}+\frac{\partial F}{\partial z}\cdot\frac{\partial z}{\partial y}=0\Rightarrow\frac{\partial z}{\partial y}=-\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial z}}$$

Def. Diferencijabilnost fje dvije i tri prom.

$f: A \to R$, $A \subseteq R^2$, $u = f(x, y)$ dif. u (x, y), $x = x(s, t)$, $y = y(s, t)$

$\Rightarrow u = f(x, y) = f(x(s,t), y(s,t))$ u (s, t) ~~ako~~ i važi:

$$\frac{\partial u}{\partial s} = \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial u}{\partial y} \cdot \frac{\partial y}{\partial s}$$

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial u}{\partial y} \cdot \frac{\partial y}{\partial t}.$$

1) Odrediti $\frac{\partial u}{\partial t}$ ako je a) $u = e^{x-2y}$, $x = \sin t$, $y = t^3$

Rj.

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial u}{\partial y} \cdot \frac{\partial y}{\partial t}$$

$$\frac{\partial u}{\partial x} = e^{x-y}, \quad \frac{\partial x}{\partial t} = \cos t, \quad \frac{\partial u}{\partial y} = -2e^{x-2y}, \quad \frac{\partial y}{\partial t} = 3t^2$$

$$\frac{\partial u}{\partial t} = e^{x-2y} \cdot \cos t - 2e^{x-2y} \cdot 3t^2 = e^{\sin t - 2t^3}(\cos t - 6t^2)$$

b) $\frac{\partial u}{\partial s}, \frac{\partial u}{\partial t} = ?$ $u = x^2 \ln y$, $x = \frac{s}{t}$, $y = 3s - 2t$

$$\frac{\partial u}{\partial s} = \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial u}{\partial y} \cdot \frac{\partial y}{\partial s} = 2x \ln y \cdot \frac{1}{t} + x^2 \frac{1}{y} \cdot 3 =$$

$$= 2 \frac{s}{t} \ln(3s - 2t) \frac{1}{t} + \frac{3s^2}{t^2 \cdot (3s - 2t)}$$

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial u}{\partial y} \cdot \frac{\partial y}{\partial t} = 2x \ln y \cdot \frac{-s}{t^2} - 2x^2 \cdot \frac{1}{y} =$$

$$= 2 \frac{s}{t} \ln(3s - 2t) \cdot \frac{-s}{t^2} - 2 \cdot \frac{s^2}{t^2} \cdot \frac{1}{3s - 2t}$$

Za $u = u(x,y,z) \Rightarrow \underbrace{\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}}_{\text{I reda}}$; $\frac{\partial^2 u}{\partial x^2}, \frac{\partial^2 u}{\partial y^2}, \frac{\partial^2 u}{\partial z^2}, \frac{\partial^2 u}{\partial x \partial y}, \frac{\partial^2 u}{\partial y \partial z}, \frac{\partial^2 u}{\partial x \partial z}$ — II reda

$$d^2u = \frac{\partial^2 u}{\partial x^2}dx^2 + 2\frac{\partial^2 u}{\partial x \partial y}dxdy + \frac{\partial^2 u}{\partial y^2}dy^2 + 2\frac{\partial^2 u}{\partial x \partial z}dxdz +$$

$$+ 2\frac{\partial^2 u}{\partial y \partial z}dydz + \frac{\partial^2 u}{\partial z^2}dz^2.$$

1) Dokazati da za fju $u = \frac{1}{\sqrt{x^2+y^2+z^2}}$ vazi: $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$

2) Za fju $u = e^{xyz}$ odrediti $\frac{\partial^3 u}{\partial x \partial y \partial z}$.

Rj:

$$\frac{\partial u}{\partial x} = e^{xyz} \cdot yz, \quad \frac{\partial u}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right) =$$

$$= \frac{\partial}{\partial y}\left| yz e^{xyz} \right| = z \cdot (e^{xyz} + y \cdot e^{xyz} \cdot xz) =$$

$$= e^{xyz}(z + xyz^2)$$

$$\frac{\partial}{\partial z}\left(\frac{\partial^2 u}{\partial y \partial x}\right) = \frac{\partial}{\partial z}\left| e^{xyz}(z + xyz^2) \right) =$$

$$= e^{xyz} \cdot xy(z + xyz^2) + e^{xyz}(1 + 2xyz)$$

5

1) Izračunati izvod implicitno zadate fje:

a) $x+y=e^{x-y}$ b) $x^2+y^2+z^2=0$

Rj:

a) $x+y=e^{x-y}$ / $\partial/\partial x$

$1+\frac{\partial y}{\partial x}=+e^{x-y}\left(1-\frac{\partial y}{\partial x}\right)$

$\frac{\partial y}{\partial x}\left(1+e^{x-y}\right)=e^{x-y}-1$

$\frac{\partial y}{\partial x}=\frac{e^{x-y}-1}{1+e^{x-y}}$

ili $F(x,y)=x+y-e^{x-y}=0$

$\frac{\partial F}{\partial x}=1-e^{x-y}$

$\frac{\partial F}{\partial y}=1+e^{x-y}$

$\frac{\partial y}{\partial x}=-\frac{1-e^{x-y}}{1+e^{x-y}}$

b) $x^2+y^2+z^2=0$

$\partial/\partial x$: $2x+2z\cdot\frac{\partial z}{\partial x}=0$

$\frac{\partial z}{\partial x}=\frac{-2x}{2z}=-\frac{x}{z}$

$\partial/\partial y$: $2y+2z\frac{\partial z}{\partial y}=0$

$\frac{\partial z}{\partial y}=-\frac{y}{z}$

2) Ispitati da li fja z def. sa $F(z-\sqrt{x},\sqrt{x}-\sqrt{y})=0$ zadovoljava j-nu: $\sqrt{x}\cdot\frac{\partial z}{\partial x}+\sqrt{y}\cdot\frac{\partial z}{\partial y}=\frac{1}{2}$.

Rj: $u=z-\sqrt{x}$, $v=\sqrt{x}-\sqrt{y}$

$\frac{\partial u}{\partial x}=-\frac{1}{2\sqrt{x}}$, $\frac{\partial u}{\partial z}=1$, $\frac{\partial v}{\partial x}=\frac{1}{2\sqrt{x}}$, $\frac{\partial v}{\partial y}=\frac{-1}{2\sqrt{y}}$, $\frac{\partial u}{\partial y}=0$, $\frac{\partial v}{\partial z}=0$

$\frac{\partial z}{\partial x}=-\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}}$, $\frac{\partial z}{\partial y}=-\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial z}}$.

$$\frac{\partial F}{\partial x}=\frac{\partial F}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial F}{\partial v}\cdot\frac{\partial v}{\partial x}=\frac{\partial F}{\partial u}\left(-\frac{1}{2\sqrt{x}}\right)+\frac{\partial F}{\partial v}\cdot\frac{1}{2\sqrt{x}}=$$

$$=\frac{\frac{\partial F}{\partial v}-\frac{\partial F}{\partial u}}{2\sqrt{x}}.$$

$$\frac{\partial F}{\partial y} = \frac{\partial F}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial F}{\partial v} \cdot \frac{\partial v}{\partial y} = \frac{\partial F}{\partial u} \cdot 0 + \frac{\partial F}{\partial v} \cdot \frac{1}{2\sqrt{y}} =$$

$$= - \frac{\frac{\partial F}{\partial v}}{2\sqrt{y}}.$$

$$\frac{\partial F}{\partial z} = \frac{\partial F}{\partial u} \cdot \frac{\partial u}{\partial z} + \frac{\partial F}{\partial v} \cdot \frac{\partial v}{\partial z} =$$

$$= \frac{\partial F}{\partial u} + 0 = \frac{\partial F}{\partial u}.$$

$$\frac{\partial z}{\partial x} = - \frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}} = - \frac{-\frac{\partial F}{\partial u} + \frac{\partial F}{\partial v}}{2\sqrt{x} \cdot \frac{\partial F}{\partial u}}$$

$$\frac{\partial z}{\partial y} = - \frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial z}} = - \frac{-\frac{\partial F}{\partial v}}{2\sqrt{y} \cdot \frac{\partial F}{\partial u}}.$$

$$\sqrt{x} \cdot \frac{\partial z}{\partial x} + \sqrt{y} \cdot \frac{\partial z}{\partial y} = \sqrt{x} \cdot \frac{\frac{\partial F}{\partial u} - \frac{\partial F}{\partial v}}{2\sqrt{x} \frac{\partial F}{\partial u}} + \sqrt{y} \cdot \frac{\frac{\partial F}{\partial v}}{2\sqrt{y} \frac{\partial F}{\partial u}} =$$

$$= \frac{\frac{\partial F}{\partial u} - \frac{\partial F}{\partial v} + \frac{\partial F}{\partial v}}{2 \frac{\partial F}{\partial u}} = \frac{1}{2}.$$